UNIT 6

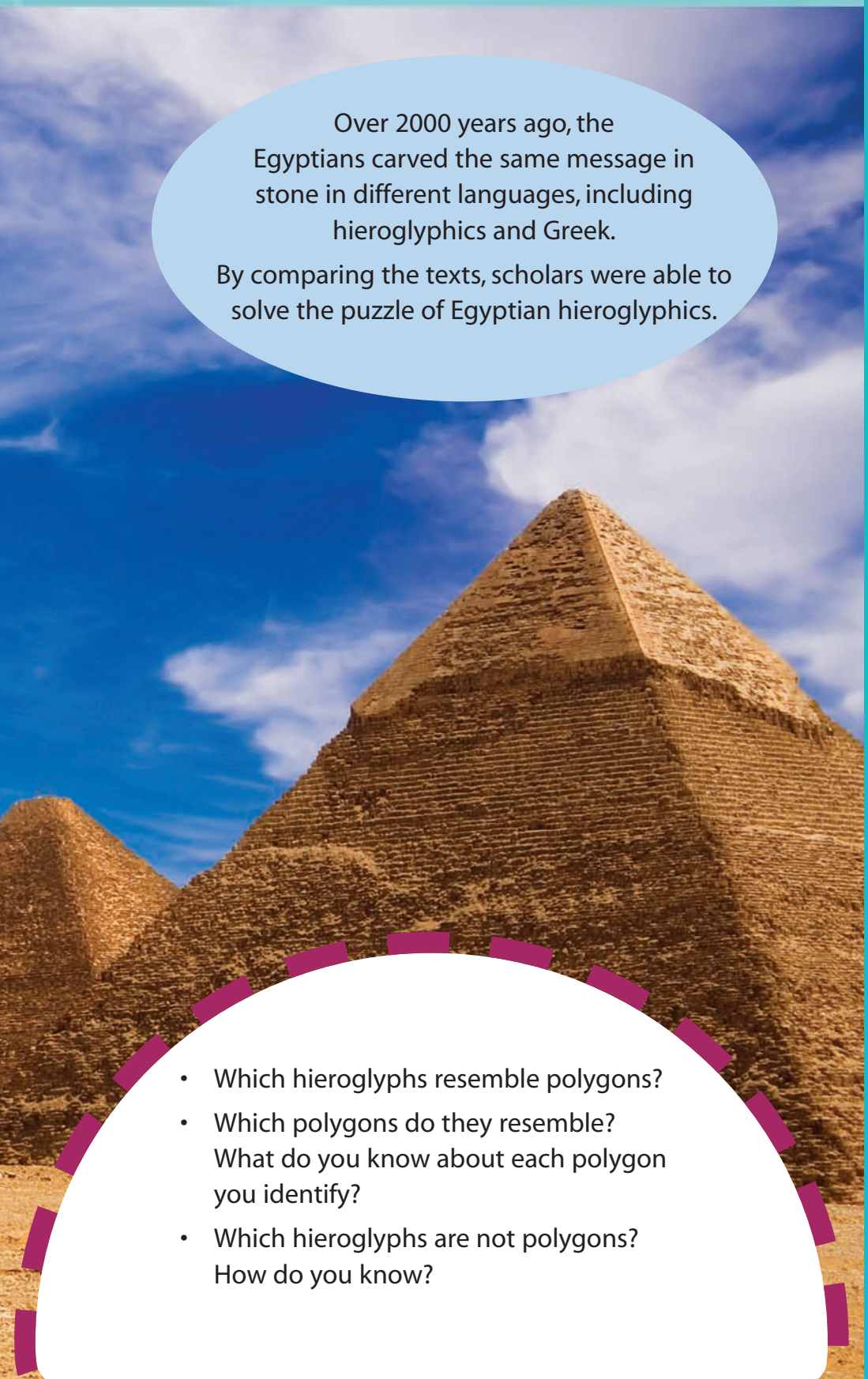
Geometry and Puzzle Mania!

Learning Goals

- construct and compare triangles
- describe and compare regular and irregular polygons
- develop formulas for the perimeters of polygons, the area of a rectangle, and the volume of a rectangular prism

These pictures are ancient Egyptian characters called *hieroglyphs*.

Measurement



Over 2000 years ago, the Egyptians carved the same message in stone in different languages, including hieroglyphics and Greek.

By comparing the texts, scholars were able to solve the puzzle of Egyptian hieroglyphics.

- Which hieroglyphs resemble polygons?
- Which polygons do they resemble? What do you know about each polygon you identify?
- Which hieroglyphs are not polygons? How do you know?

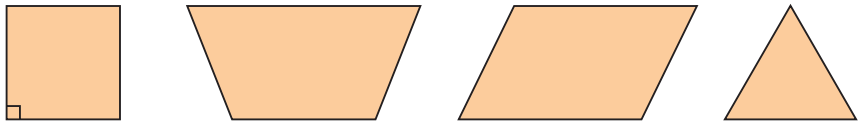
Key Words

- equilateral triangle
- isosceles triangle
- scalene triangle
- acute triangle
- right triangle
- obtuse triangle
- non-polygon
- regular polygon
- irregular polygon
- convex polygon
- concave polygon
- congruent
- formula

LESSON 1

Exploring Triangles

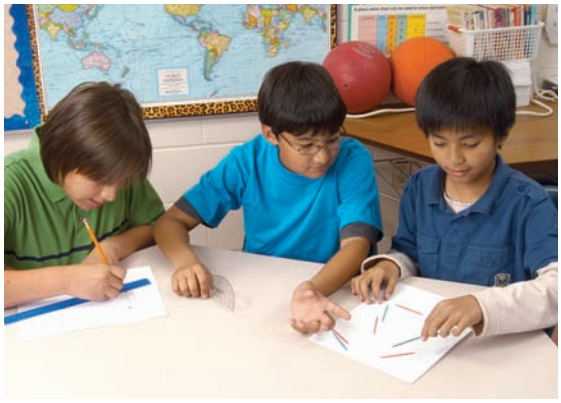
Which sorting rules can you use to sort these shapes?



Explore

You will need 9 toothpicks, a ruler, a protractor, and scissors.

- Use at most 9 toothpicks. Make a triangle on paper.
- Mark a dot at each vertex of the triangle.
- Remove the toothpicks.
  Use a ruler to draw the triangle.
- Are there any equal sides? Equal angles?
  Record your findings.
- Repeat the activity to draw at least
  5 different triangles. Cut out the triangles.
- Choose a sorting rule. Sort the triangles.



Show *and* Share

Trade your sorted triangles with another group of students.
Identify the rule for your classmates' sorting.
Did you use the same rule to sort? Explain.
What else do you notice about the triangles?

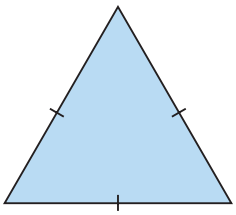
Connect

- We can:
  - Use a ruler to measure the side lengths of a triangle.
  - Use a protractor to measure the angles in a triangle.
  - Fold a triangle, or use a Mira to find the lines of symmetry in a triangle.

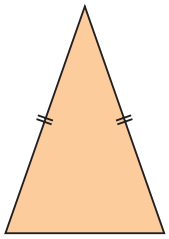
So, we can use these attributes to sort triangles.

➤ We can name triangles according to how their side lengths compare.

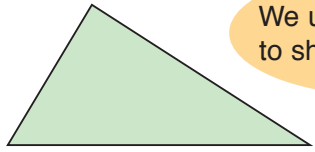
An **equilateral triangle** has 3 equal sides.



An **isosceles triangle** has 2 equal sides.



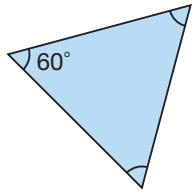
A **scalene triangle** has no equal sides.

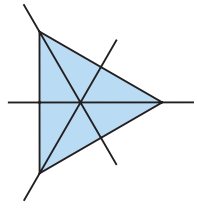


We use hatch marks to show equal sides.

➤ Here are some other attributes of triangles.

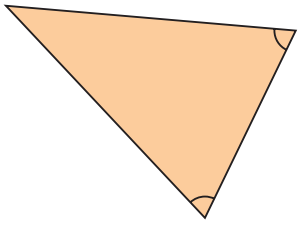
- An equilateral triangle has 3 equal angles and 3 lines of symmetry.
  Since the sum of the angles in a triangle is 180°, each angle measure is: $180° \div 3 = 60°$
  All equilateral triangles have angle measures of 60°.

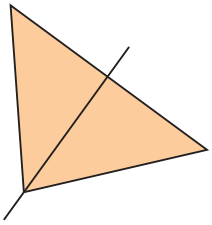




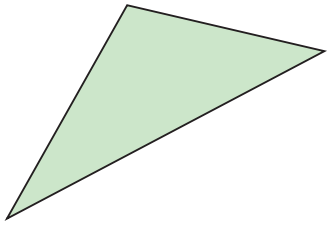
We use matching arcs to show equal angles.

- An isosceles triangle has 2 equal angles and 1 line of symmetry.





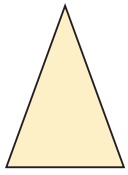
- A scalene triangle has no equal angles and no lines of symmetry.



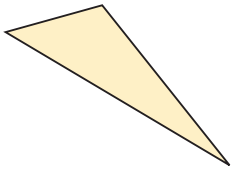
Practice

1. Name each triangle as isosceles, equilateral, or scalene.
How did you decide which name to use?

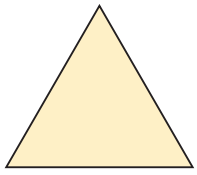
a)



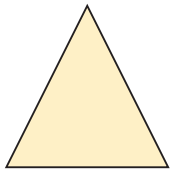
b)



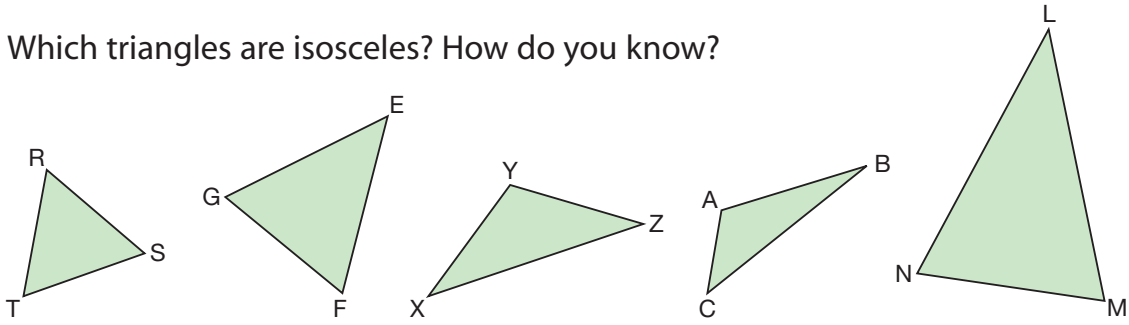
c)



d)



2. **a)** Which triangles are isosceles? How do you know?



b) For each isosceles triangle, name the sides that have the same length, and the angles that have the same measure.

c) Which triangle is equilateral? How do you know?

d) Which triangle is not isosceles and not equilateral? Which type of triangle is it?

3. Use a geoboard, geobands, and square dot paper.

a) Make 3 different scalene triangles.
Record each triangle on dot paper.
How do you know each triangle is scalene?

b) Make 3 different isosceles triangles.
Record each triangle on dot paper.
How do you know each triangle is isosceles?

c) Try to make an equilateral triangle.
What do you notice?

4. Work with a partner.

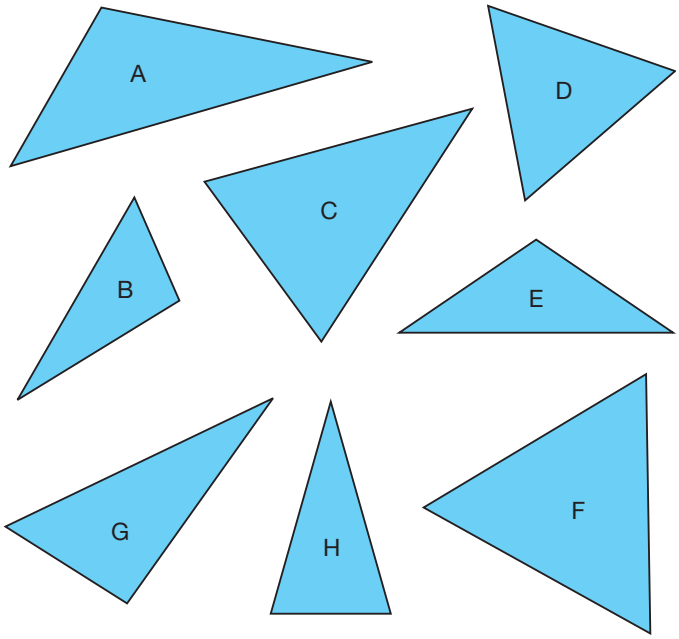
a) Look around you. Find 2 examples of:
- a scalene triangle
- an isosceles triangle
- an equilateral triangle

Sketch each triangle. Describe where you found it.

b) Which type of triangle was easiest to find? Why might this be?

5. Here is the truss of the Burrard Street Bridge in Vancouver, BC.
Which types of triangles do you see in the truss? How could you check?

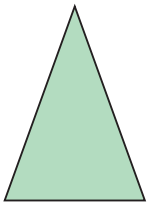
6. Your teacher will give you a large copy of these triangles.



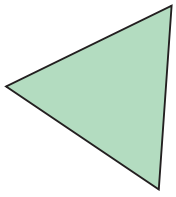
a) List the attributes of each triangle.
b) Sort the triangles by the number of equal sides.
c) Sort the triangles by the number of equal angles.
d) What do you notice about your sortings?

7. Identify each triangle as equilateral, isosceles, or scalene.
Which strategy did you use?

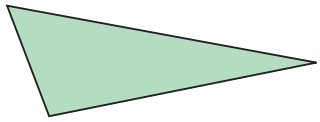
a)



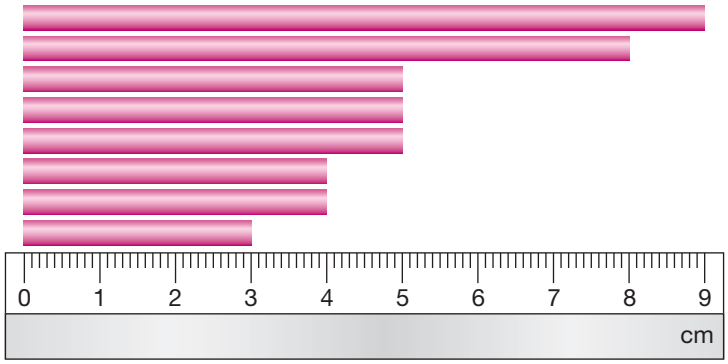
b)



c)



8. You will need drinking straws, a ruler, scissors, and pipe cleaners.
Cut the straws into 8 pieces as shown. Use pieces of pipe cleaner as joiners.

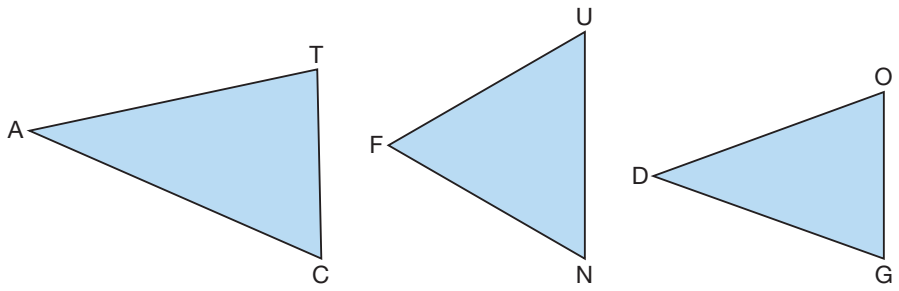


a) Make each triangle.
Trace and label your results.
- an equilateral triangle
- an isosceles triangle with the least perimeter
- a scalene triangle with the greatest perimeter

b) Which straws could not be used together to make a triangle? Explain.

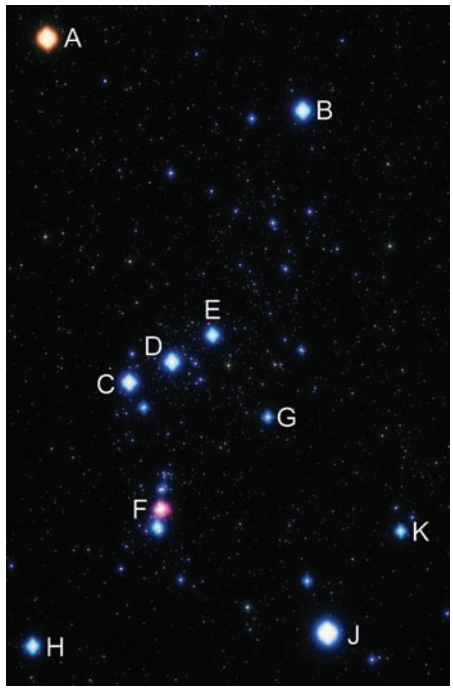
Perimeter is the distance around a shape.

9. **a)** Name each triangle as scalene, isosceles, or equilateral.
Explain your choice each time.



b) How can you use the measures of the angles in a triangle to predict how the lengths of the sides compare?

10. Your teacher will give you a copy of this picture of the Orion constellation.
The brightest stars are labelled with letters.



a) Connect points C, D, and F to form a triangle.
Which type of triangle did you form?
How do you know?

b) Connect points F, H, and J to form a triangle.
Which type of triangle did you form?
How do you know?

c) Which points would you connect to form an equilateral triangle?
Check by measuring the angles.

11. Use a geoboard, geobands, and square dot paper.

a) Make an isosceles triangle.
Draw the triangle on dot paper.

b) Use the triangle from part a.
Change the triangle so it is scalene.
Describe the changes you made.

Reflect

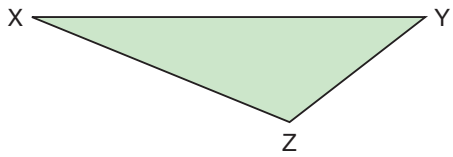
Explain how you remember how many equal sides each of these triangles has.

- an equilateral triangle
- an isosceles triangle
- a scalene triangle

LESSON 2

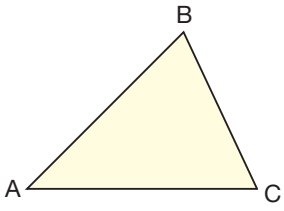
Naming and Sorting Triangles by Angles

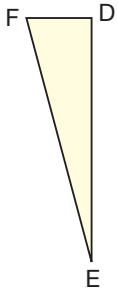
Name each angle in △XYZ as acute, right, or obtuse.
What strategy did you use to find out?
What is the sum of the angles in the triangle?

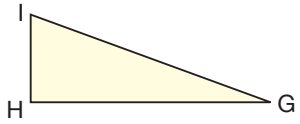


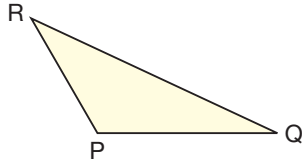
Explore

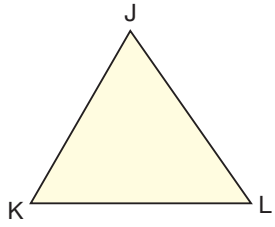
You will need a protractor and scissors.
Your teacher will give you a large copy of these triangles.

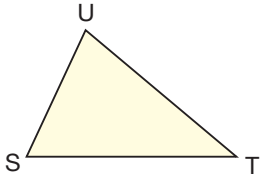


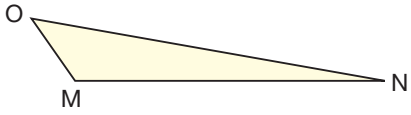


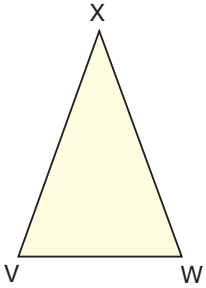












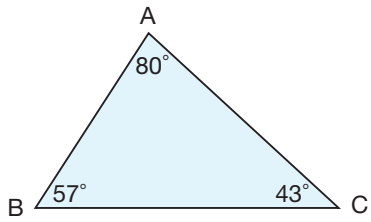
- Measure the angles in each triangle.
  Record the angle measures.
- Cut out the triangles. Choose a sorting rule, then sort the triangles.
  How are the triangles in each group the same? How are they different?

Show *and* Share

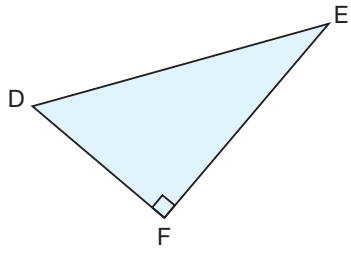
Trade your sorted triangles with another group of students.
Identify the rule for your classmates' sorting.
Did you sort the triangles the same way? Explain.

Connect

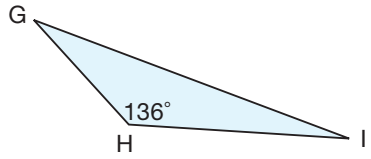
➤ We can name triangles by the types of interior angles.
An **acute triangle** has all angles less than 90°.



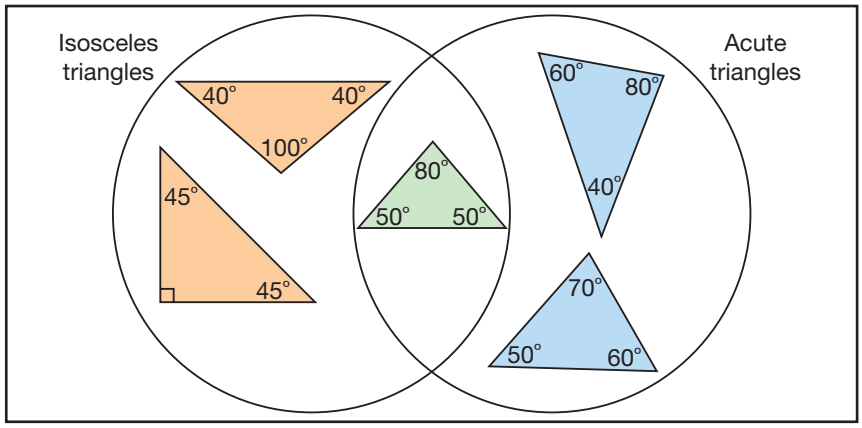
A **right triangle** has one 90° angle.



An **obtuse triangle** has one angle greater than 90°.

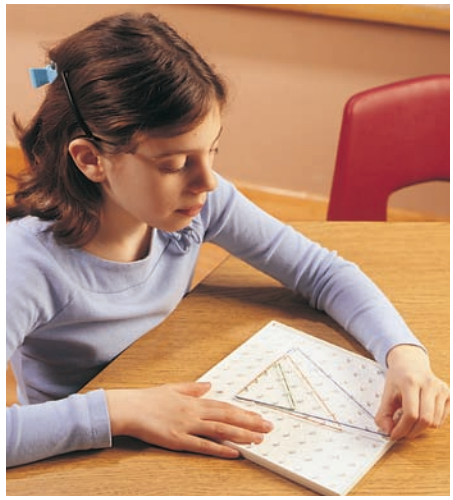


➤ We can sort triangles in a Venn diagram.
For example, choose the sorting rule "Isosceles triangles" and "Acute triangles."



The triangles in the left loop have 2 equal angles.
The triangles in the right loop have all angles less than 90°.
The triangle in the overlap has 2 equal angles and all angles less than 90°.

Practice



1. Use a geoboard, geobands, and square dot paper.

a) Make 3 different acute triangles.
Draw each triangle on dot paper.
How do you know each triangle is acute?

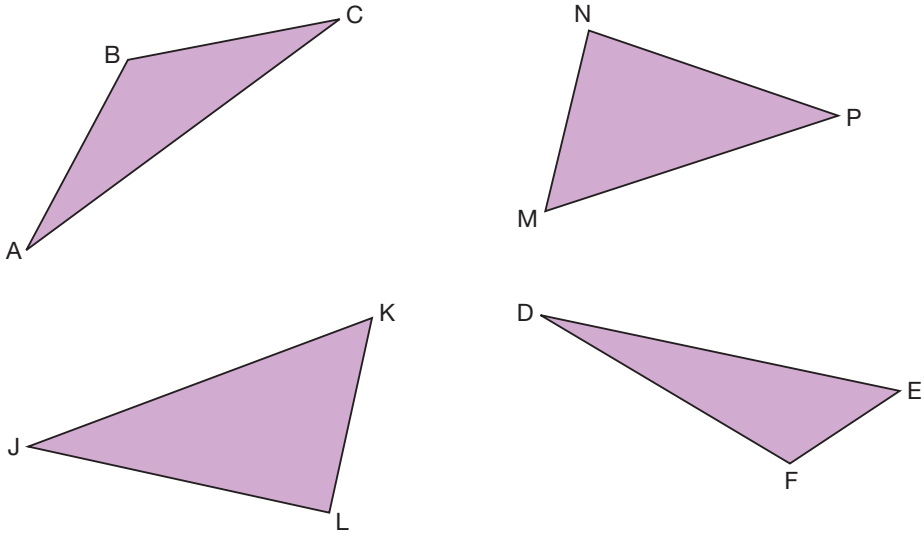
b) Make 3 different obtuse triangles.
Draw each triangle on dot paper.
How do you know each triangle is obtuse?

c) Make 3 different right triangles.
Draw each triangle on dot paper.
How do you know each triangle is right?

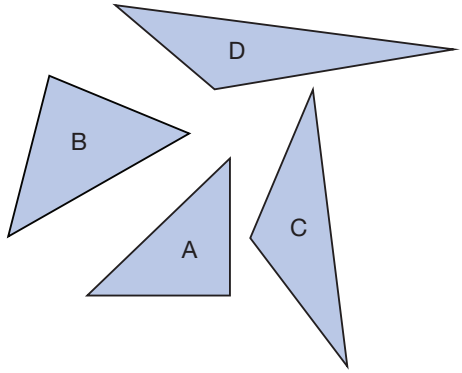
2. **a)** Predict whether each triangle is an acute, an obtuse, or a right triangle.
How did you make your prediction?

b) Use a protractor. Measure the angles in each triangle.
Name each triangle as an acute, an obtuse, or a right triangle.

c) Were your predictions correct? Explain.



3. Akna drew these triangles. He noticed there were at least two acute angles in each triangle he drew.
Akna made this conclusion: "All triangles must have at least two acute angles."
Do you agree?
Why or why not?



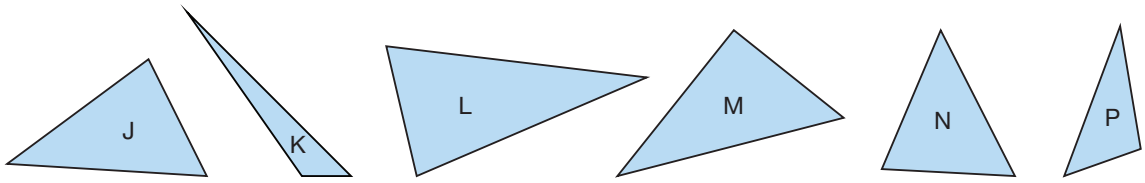
4. Is each statement true or false?
Use pictures, words, or numbers to explain your thinking.

a) A triangle can have more than one obtuse angle.

b) A triangle can have only one 90° angle.

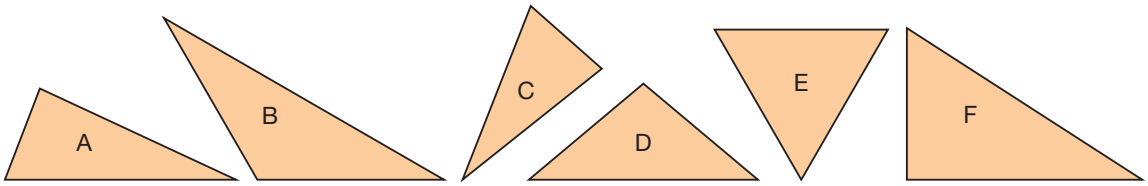
c) A triangle can have 3 acute angles.

5. You will need scissors and a large copy of these triangles.



Cut out the triangles.
Sort the triangles as acute, obtuse, or right triangles.
How did you decide where to place each triangle?

6. You will need scissors and a large copy of these triangles.
Cut out the triangles.



a) Sort the triangles in a Venn diagram with 2 loops.
Label each loop. Explain your sorting rule.
Are there any triangles in the overlap?
If there are, what attributes do these triangles have?

b) Repeat part a. This time, choose a different sorting rule.
How many different ways can you sort the triangles? Show your work.

7. Sort the triangles in question 6 using a Venn diagram with 3 loops.
Record your work. Do any of the loops overlap?
Why or why not?

8. **a)** Can an obtuse triangle be an equilateral triangle? Explain.

b) Can a right triangle be an isosceles triangle? Explain.

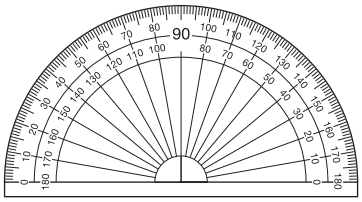
Reflect

How many different ways can you describe a triangle?
Draw a triangle and describe it as many ways as you can.

LESSON 3

Drawing Triangles

We can use a protractor to draw an angle.
What steps would you take to draw a 45° angle?



Explore

You will need rulers and protractors.

➤ Each group member chooses 2 triangles from the list:
- acute
- obtuse
- right
- scalene
- isosceles
- equilateral

➤ Draw each triangle you chose.

➤ Trade triangles with another group member.
Identify each triangle.

Show *and* Share

Compare your strategies for drawing with those of the others in your group.
How did you create each triangle?
How did you identify your group members' triangles?

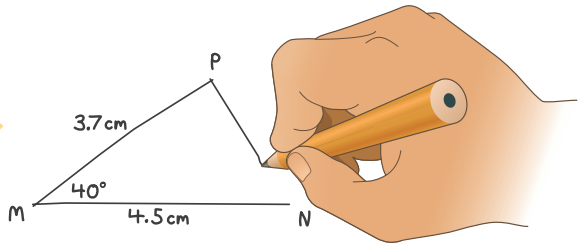
Connect

We can use a ruler and a protractor to construct a triangle.

Construct scalene △MNP.
The length of MN is 4.5 cm.
The measure of ∠M is 40°.
The length of MP is 3.7 cm.

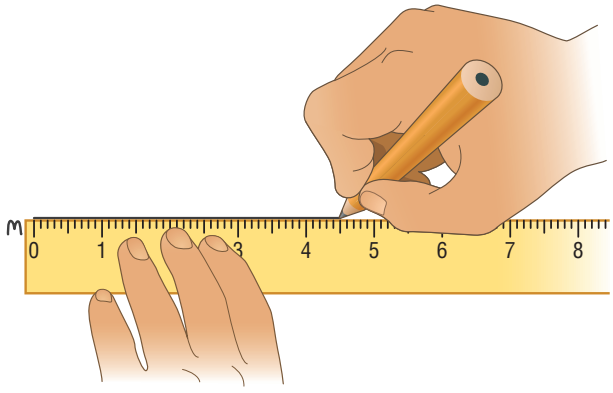
Step 1

Sketch the triangle first.
Label each side and angle.
This sketch is *not* accurate.



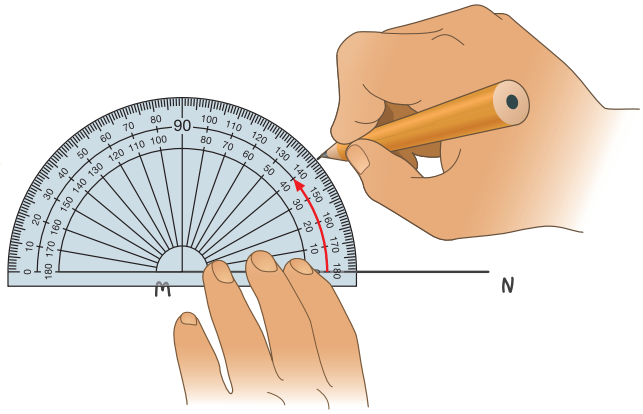
Step 2

Use a ruler to draw side MN
4.5 cm long.

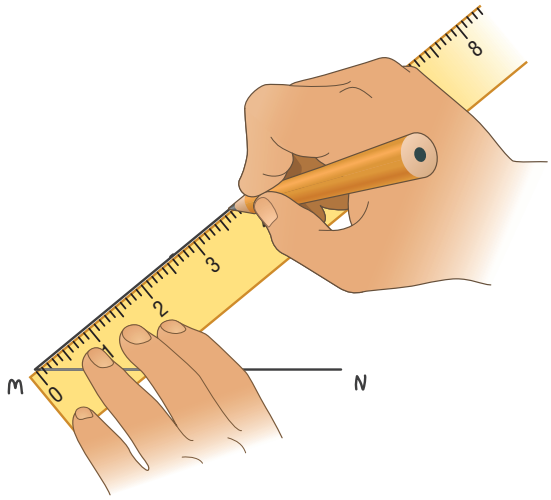


Step 3

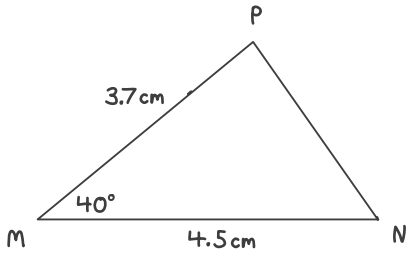
Place the baseline of the protractor
on MN, with its centre at M.
From 0° on the inner circle,
measure an angle of 40° at M.



Step 4
Remove the protractor.
Join M to the mark at 40°.
Measure 3.7 cm from M.
Mark the point P.



Step 5
Use a ruler to join P to N
to form side NP.
Label the triangle with its measures.



Practice

1. Use either or both of these tools: ruler and protractor
   - Construct each triangle listed below.
   - Explain how you know you have drawn that triangle.

   a) an acute triangle
   b) an equilateral triangle
   c) an isosceles triangle
   d) an obtuse triangle
   e) a right triangle
   f) a scalene triangle

2. Use a ruler and a protractor.
   Construct a triangle with angles 40°, 60°, and 80°.
   Compare your triangle with that of a classmate.
   Do your triangles match?
   How could you find out?

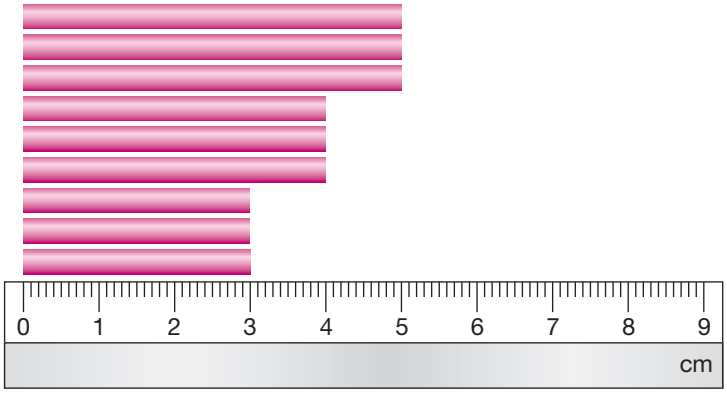
3. Use a ruler and a protractor.
Construct each triangle.
Sketch the triangle first.

a) Isosceles triangle VWX
The length of side VW is 7 cm.
The measure of ∠V is 80°.
The measure of ∠W is 50°.

b) Obtuse triangle RST
The length of side TS is 5.2 cm.
The measure of ∠T is 30°.
The length of side RT is 3.4 cm.

Label each triangle with the measures of all the sides and angles.

4. You will need drinking straws, a ruler, scissors, and pipe cleaners.
Cut the straws into 9 pieces as shown.



Use pieces of pipe cleaner as joiners.
Use combinations of 3 or more straws to make each triangle.
Trace each triangle.
Label each triangle with the measures of all the sides and angles.

a) an isosceles triangle that is also an acute triangle
b) an isosceles triangle that is also an obtuse triangle
c) two different equilateral triangles
d) two different right triangles

5. Use a geoboard and geobands.
Construct a triangle with two 45° angles.
Record your work on square dot paper.
Do this 3 times to construct 3 different triangles.

a) How are the triangles the same?
How are the triangles different?

b) What kind of triangle did you make?
Give a different name to describe the triangle.

6. Construct a triangle that has one angle that measures 55° and one angle that measures 35°.
What kind of triangle did you make?
Give a different name to describe the triangle.

7. Construct a triangle that has one angle that measures 60° and one angle that measures 45°.

a) What is the measure of the third angle?

b) What kind of triangle did you make?
How do you know?

c) How else can you name the triangle?

8. A student said he had drawn △ABC with these measures:

- AB = 4.2 cm
- ∠A = 90°
- ∠B = 95°

Was the student correct?
How do you know?

9. Construct isosceles △GHK.
The measure of ∠H is 120°.
Choose side lengths for HG and HK so that △GHK is isosceles.

a) What are the measures of ∠G and ∠K?
How long is side GK?

b) Suppose side HG is longer.
The length of side HK does not change.
What happens to the measure of ∠K?
What happens to the length of side GK?

Show your work.

Reflect

Name the 6 types of triangles you know.
Which of them do you find easiest to draw?
Explain why.

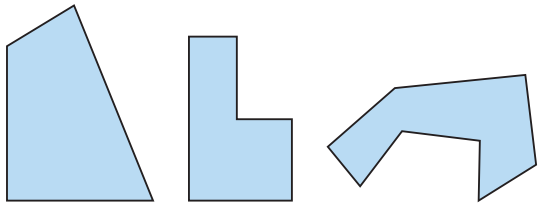
At Home

Look for triangles in your home.
They could be pictures of triangles or objects with triangular faces.
Name each triangle 2 ways.
Choose 1 triangle. Draw it.

LESSON 4

Investigating Polygons

What do we call a polygon with 4 sides?
With 6 sides? With 8 sides?

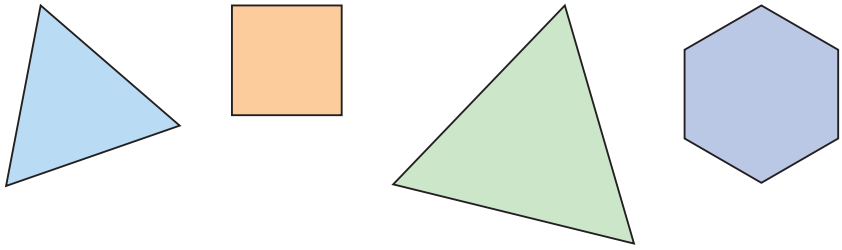


Explore

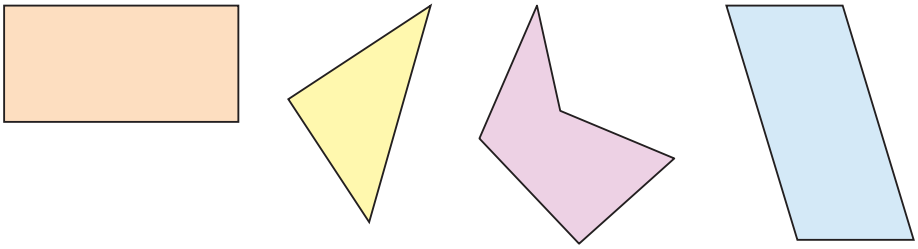
You will need a ruler and a protractor.
Your teacher will give you a large copy of these shapes.

Mystery Sort!

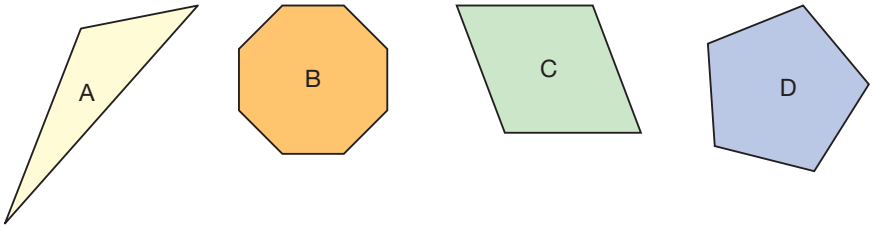
Set 1: All of these shapes have the same attribute.



Set 2: None of these shapes has that attribute.



Set 3: Which of these shapes have that attribute?



Which attribute do the shapes in Set 1 share?

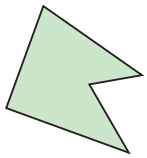
Show *and* Share

Share your results with another pair of students.
Did you find the same attribute?
If not, check that both attributes are correct.
How did you decide which shapes in Set 3 have the attribute?
Which other shapes could you place in Set 1? Explain.

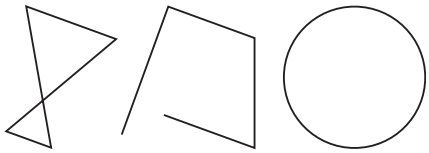
Connect

A polygon is a closed shape with sides that are straight line segments.
Exactly 2 sides meet at a vertex. The sides intersect only at the vertices.

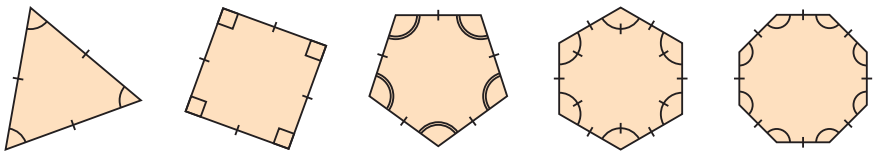
This shape is a polygon.



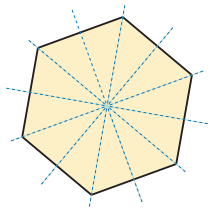
These shapes are **non-polygons**.



A **regular polygon** has all sides equal and all angles equal.
These polygons are regular.



A regular polygon has line symmetry.
A regular hexagon has 6 lines of symmetry.

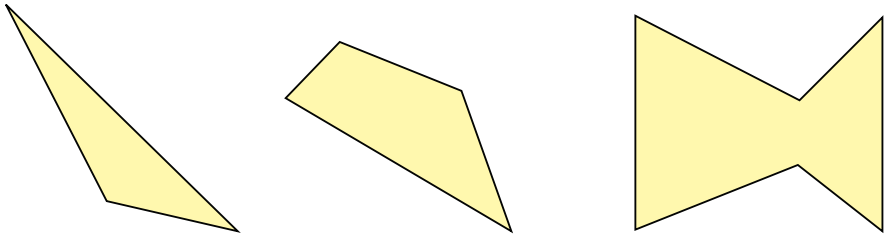


Math Link

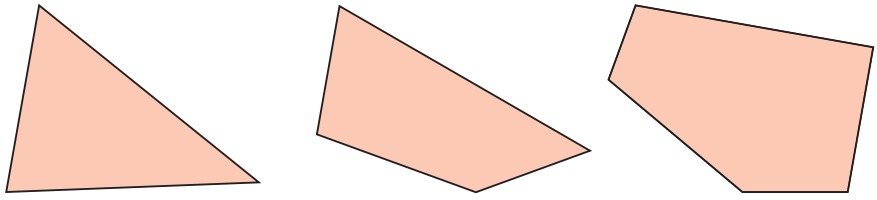
Your World

A *parfleche* is a container used by the Plains people to carry dried meat, clothing, tools, and other goods. It is usually made from buffalo hide, then painted with a design. The design represents a particular band, and is passed down from generation to generation. Which polygons do you see in the design on this parfleche of the Crow Nation?

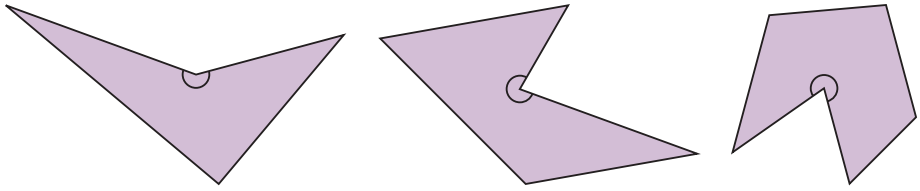
An **irregular polygon** does not have all sides equal and all angles equal.
These polygons are irregular.



A **convex polygon** has all angles less than 180°.
These polygons are convex.



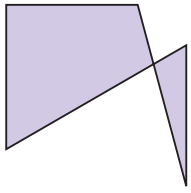
A **concave polygon** has at least one angle greater than 180°.
These polygons are concave.



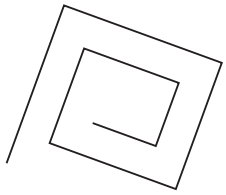
Practice

1. Explain why each shape is not a polygon.

a)

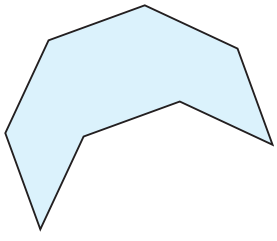


b)

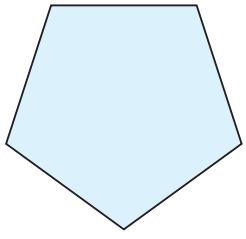


2. Is each polygon regular? How do you know?

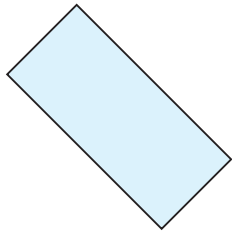
a)



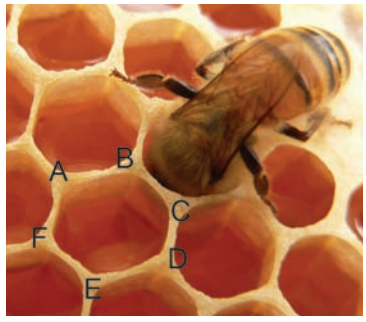
b)



c)



3. A cell in a honeycomb approximates a regular hexagon.

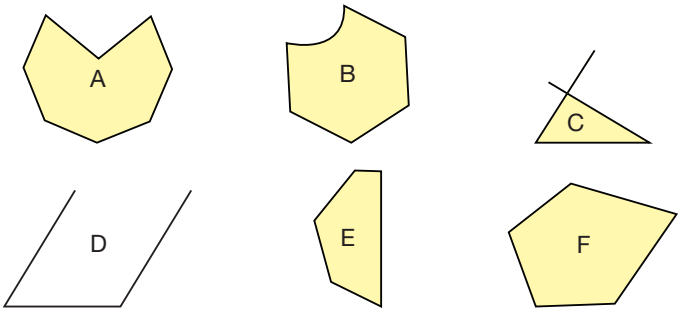


a) Suppose $\angle A = 120°$. What are the measures of angles B, C, D, E, and F?

b) Suppose side AB has length 9 cm.
What are the lengths of sides BC, CD, DE, EF, and FA?

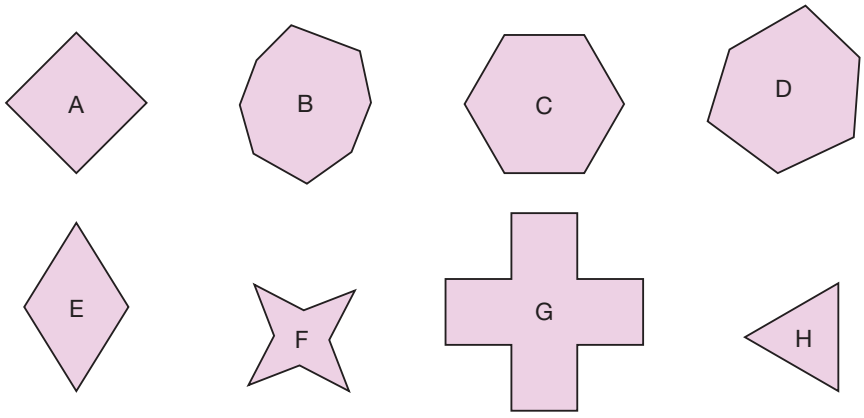
4. Your teacher will give you a large copy of these shapes.

a) Sort these shapes into sets of polygons and non-polygons. Explain how you decided where to place each shape.



b) Draw a different shape that belongs in each set.
Explain how you know that it belongs.

5. Your teacher will give you a large copy of these polygons.



a) Which polygons appear to be regular?

b) How can you check that the polygons you identified in part a are regular? Use your strategy to check.

c) Sort the polygons into sets of regular and irregular polygons.

d) For each set in part c, draw a different polygon that belongs in that set.

e) Sort the polygons into sets of convex and concave polygons.

f) For each set in part e, draw a different polygon that belongs in that set.

6. Your teacher will give you a large copy of these road signs.

a) Name the polygon that each sign reminds you of.

b) Sort the signs into sets of regular and irregular polygons.
Explain how you did this.

7. **a)** Find at least 3 different irregular polygons outside the classroom.
Describe each polygon you find.

b) Find at least 3 different regular polygons outside the classroom.
Describe each polygon you find.
Name each polygon.

8. **a)** What do we call:
- a regular triangle?
- a regular quadrilateral?

b) Use dot paper.
Draw 3 different regular triangles.
Draw 3 different regular quadrilaterals.

c) What do you notice about the regular triangles you drew?
What do you notice about the regular quadrilaterals you drew?

9. Can a concave quadrilateral be regular?
Explain.

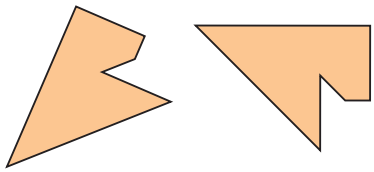
Reflect

List the attributes of a regular polygon.
Which strategy do you prefer to use to check whether a polygon is regular or irregular?
Explain your choice.

LESSON 5

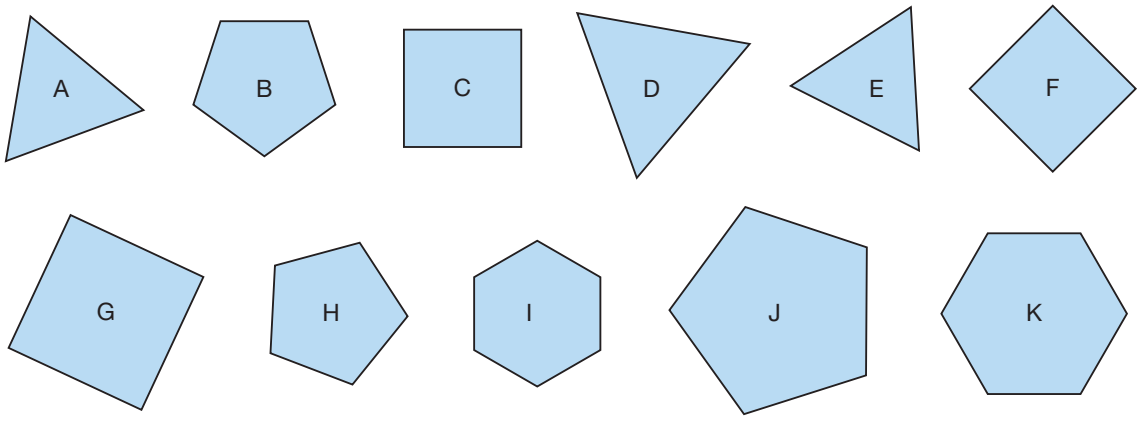
Congruence in Regular Polygons

Do these shapes match?
How could you find out?



Explore

You will need tracing paper, a protractor, and a millimetre ruler.
Your teacher will give you a large copy of these polygons.



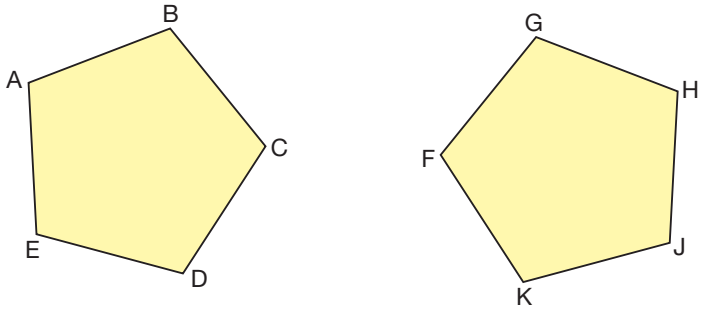
- Identify pairs of polygons that match.
  How do you know that they match?
- Choose a pair of matching polygons.
  Measure and record their side lengths.
  Measure and record their angles.
  Repeat these measures for other pairs of matching polygons.
- What do you notice about the side lengths and angle measures of matching polygons? Explain.

Show *and* Share

Share your work with another pair of students.
Check that you found the same pairs of matching polygons.
What other strategy could you use to tell if two polygons match?

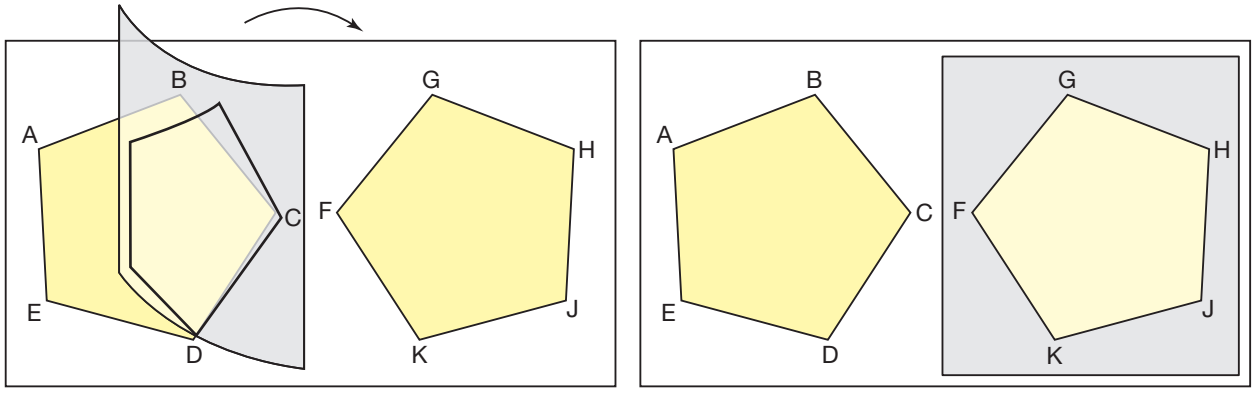
Connect

When polygons match exactly, the polygons are **congruent**.
Here are two ways to show that these pentagons are congruent.

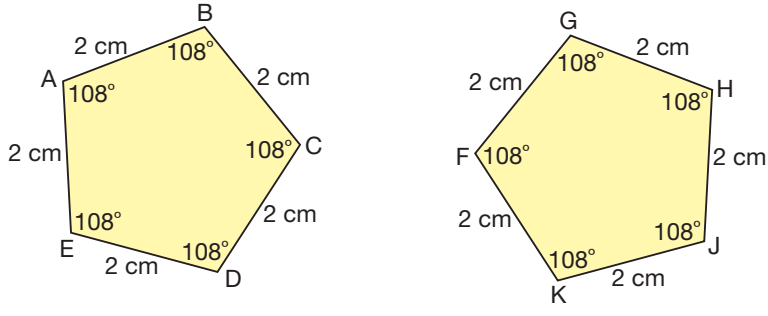


➤ Place one pentagon on top of the other.
If they match exactly, they are congruent.
You may need to flip or turn the shapes to show they are congruent.
If you cannot move the pentagons:
Trace one pentagon, then place the tracing on top of the other pentagon.

When one shape is placed on top of another and the two shapes match exactly, we say they *coincide*. One shape is *superimposed* on the other.



➤ Measure and record the lengths of all the sides.
Measure and record all the angle measures.

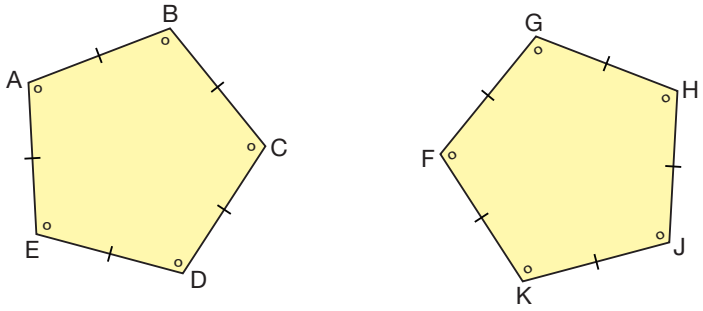


Compare the measures.

All sides have the same length.
AB = BC = CD = DE = EA = FG = GH = HJ = JK = KF

All angles have the same measure.
$\angle A = \angle B = \angle C = \angle D = \angle E = \angle F = \angle G = \angle H = \angle J = \angle K$

We use the word *congruent* to describe equal sides and equal angles.



Use hatch marks and symbols to show the equal sides and equal angles.

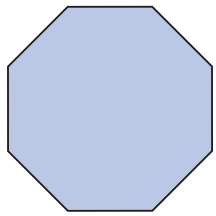
In pentagons ABCDE and FGHJK, all sides are equal and all angles are equal.
So, the pentagons are congruent.

Since all sides and angles are equal, we start with any vertex, then write the vertices in a clockwise or counterclockwise order.

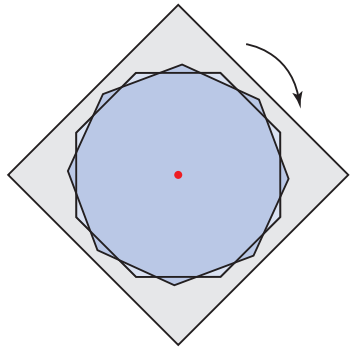
We say: "Pentagon ABCDE is congruent to pentagon FGHJK."
We write: ABCDE ≅ FGHJK

The symbol ≅ means "is congruent to."

Here is a regular octagon.
We can use a tracing of the octagon to show that all sides are equal and all angles are equal.



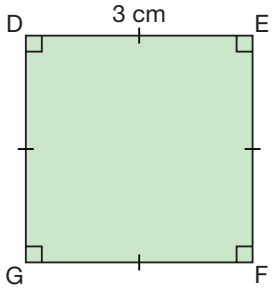
➤ Trace the octagon.
Place the tracing to coincide with the octagon.
Each angle in the tracing fits exactly over an angle in the original octagon.
Each side in the tracing fits exactly over a side in the original octagon.

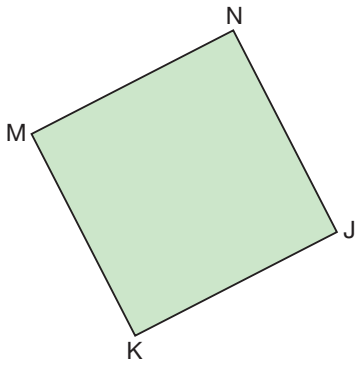


Rotate the tracing until the octagons coincide again.
Keep rotating until you have checked every side and every angle.
Then you know that all the angles are congruent, and all the sides are congruent.

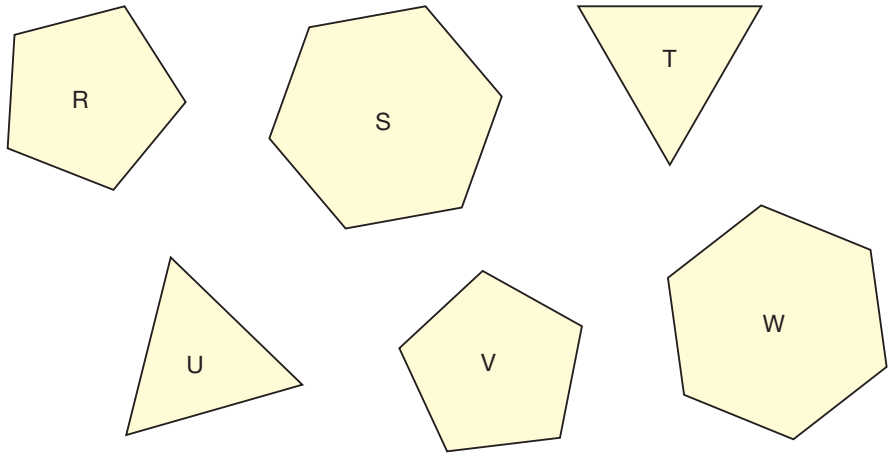
Practice

1. Quadrilaterals DEFG and JKMN are congruent.
   a) Without using a protractor, write the measure of each angle in JKMN.
   b) Without using a ruler, write the length of each side in JKMN.

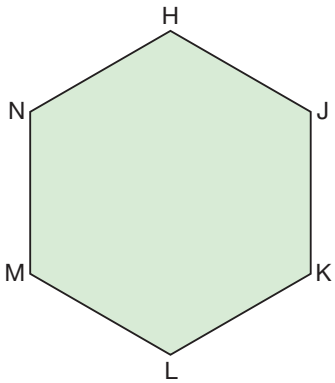




2. Which of these polygons are congruent?
   How can you tell?

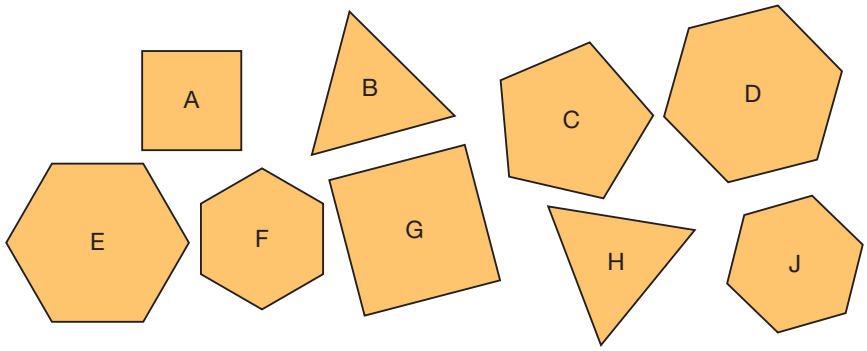


3. a) Use tracing paper. Trace hexagon HJKLMN on paper.
   Label the vertices of the traced hexagon UVWXYZ.



   b) Find the side lengths and angle measures of both hexagons.
   What do you notice?

4. Your teacher will give you a large copy of these polygons.
Use whatever materials you need.



a) Which pairs of polygons have corresponding angles congruent?
Which strategy did you use to find out?

b) Which pairs of polygons have corresponding sides congruent?
Which strategy did you use to find out?

c) Which pairs of polygons in parts a and b are congruent?
How did you decide?

Show your work.

5. Work with a partner. You will need tracing paper and a ruler.
Each of you draws a triangle. Use tracing paper to draw
2 exact copies of the triangle in different orientations.
Trade triangles with your partner.
Check that your partner's triangles are congruent.
Which strategy did you use to check?

6. Draw a regular hexagon on triangular dot paper.
Use measuring and superimposing to show that all
angles are congruent and all sides are congruent.
Show your work.

7. A student drew a rectangle on grid paper.
The student said, "Since all the angles measure 90°, the angles are
congruent. So, the rectangle is a regular quadrilateral."
Do you agree? Why or why not?

Reflect

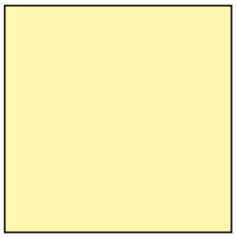
What does it mean when we say two regular polygons are congruent?
Include diagrams in your explanation.

LESSON 6

Strategies Toolkit

Explore

You will need square dot paper.
Cerise and René have a square.
They draw diagonals to divide the square into triangles.
How many triangles will they make?
Are any of the triangles congruent? Explain.
Which types of triangles are made?

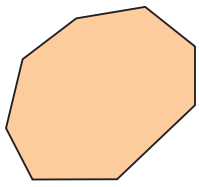


Show *and* Share

Describe the strategy you used to solve the problem.

Connect

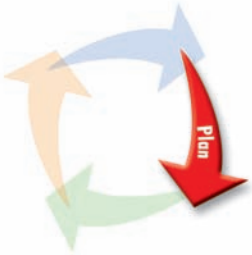
➤ Josette has a convex octagon.
She draws all of its diagonals.
How many diagonals
did Josette draw?



What do you know?

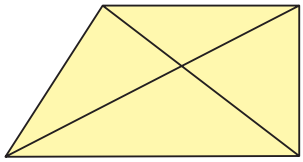
- An octagon has 8 sides.
- A diagonal is a line segment that joins 2 vertices of a polygon, but is not a side of the polygon.

Think of a strategy to help you solve the problem.



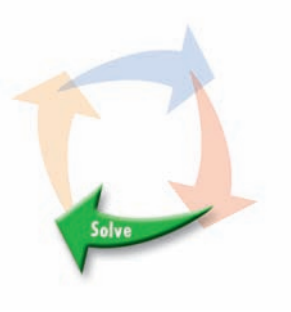
- You can **solve a simpler problem**, then **extend a table**.

➤ Draw a convex quadrilateral.
Draw its diagonals.
Two diagonals are drawn.

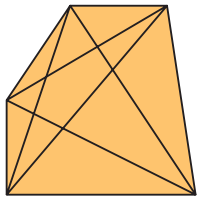


Strategies

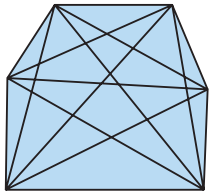
- **Make a table.**
- **Solve a simpler problem.**
- **Guess and test.**
- **Make an organized list.**
- **Use a pattern.**



➤ Draw a convex pentagon.
Draw its diagonals.
Five diagonals are drawn.



➤ Draw a convex hexagon.
Draw its diagonals.
Nine diagonals are drawn.



➤ Record your work in a table.

Shape	Number of Sides	Number of Diagonals
Quadrilateral	4	2
Pentagon	5	5
Hexagon	6	9

How many diagonals are in an octagon?
Extend the pattern in the number of diagonals column.
To check your answer, sketch the octagon.
Then draw the diagonals.

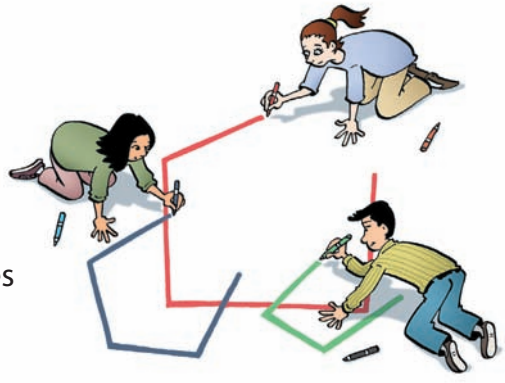
Practice

Choose one of the **Strategies**

1. Draw three different polygons.
Each polygon should have 5 diagonals.
Which strategy did you use?

2. Draw a polygon with 2 diagonals
so that the triangles formed are:
   - 4 congruent right triangles
   - 2 pairs of congruent isosceles triangles

What shape have you drawn?



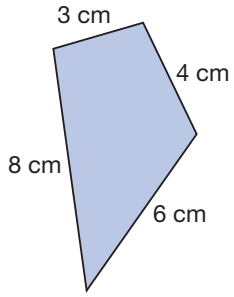
Reflect

Choose one of the *Practice* questions above.
Describe how you solved it.

LESSON 7

Perimeters of Polygons

What is the perimeter of this quadrilateral?



Explore

You will need geoboards, geobands, dot paper, and rulers.
Share the work.

Make 15 different polygons.
Make sure there are at least two of each of these types of polygons:

- square
- rectangle
- parallelogram
- rhombus
- triangle

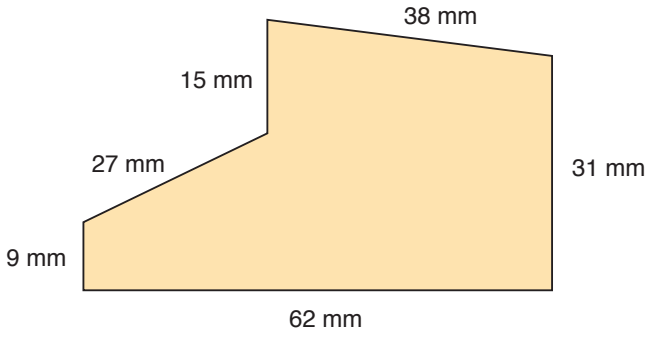
Record each polygon on dot paper.
Find the perimeter of each polygon.
For which types of polygons can you write a rule to calculate the perimeter? Write these rules.

Show *and* Share

Share your rules with another group of students.
Compare your rules. Discuss any differences.
For which types of polygons is it possible to write more than one rule? Explain.

Connect

Perimeter is the distance around a polygon.
You discovered that we can use rules to find the perimeter of polygons.
For this hexagon:

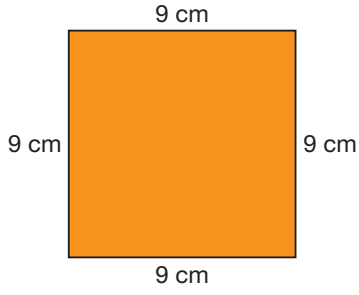


Perimeter $= 38 + 31 + 62 + 9 + 27 + 15$
$= 182$
The perimeter of this hexagon is 182 mm.

Our rule is, for any polygon, we can find the perimeter by adding the side lengths.

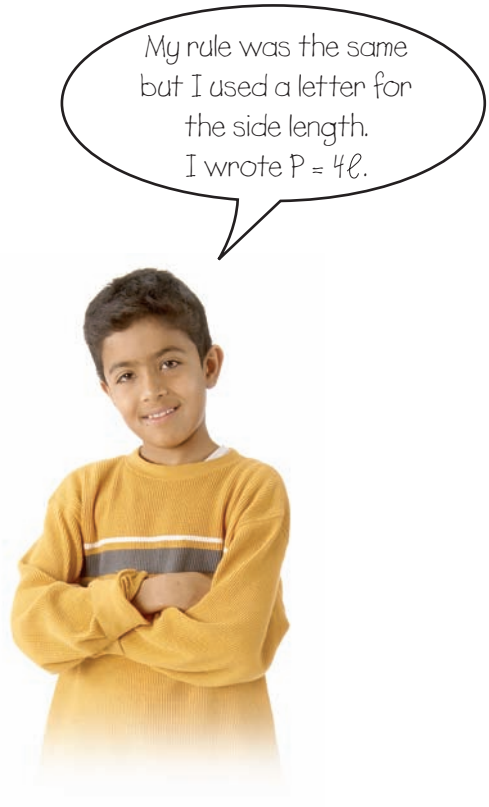
We can also develop rules that apply to specific polygons.

➤ Here is Katy's way to find the perimeter of this square.

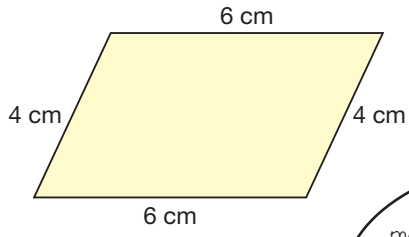


Perimeter $= 9 + 9 + 9 + 9$
$= 4 \times 9$
$= 36$
The perimeter of this square is 36 cm.

A square has 4 equal sides.
Katy says this suggests a rule for finding the perimeter of any square:
Multiply the side length by 4.



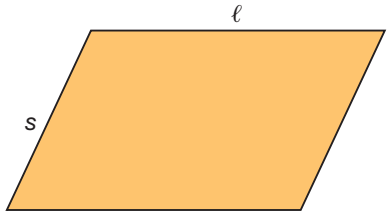
➤ Here is Graeme's way to find the perimeter of this parallelogram.



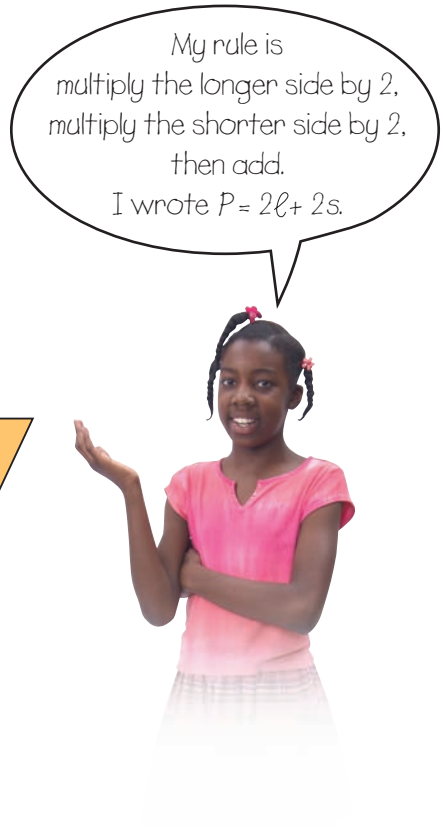
Perimeter $= 6 + 4 + 6 + 4$
$= (6 + 4) + (6 + 4)$
$= 2 \times (6 + 4)$
$= 2 \times 10$
$= 20$

The perimeter of this parallelogram is 20 m.

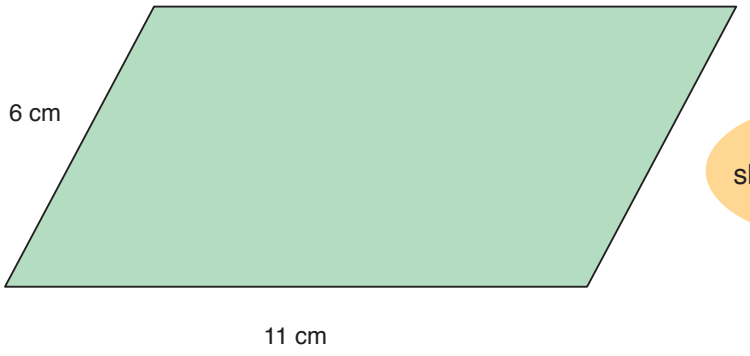
A parallelogram has two pairs of congruent sides.
Graeme says this suggests a rule for finding the perimeter of any parallelogram:
Add the measures of a longer side and a shorter side, then multiply by 2.



A rule for finding the perimeter of any parallelogram is:
Perimeter $= 2 \times (\ell + s)$



➤ We can use these **formulas** to find the perimeter of the parallelogram below.



A *formula* is a short way to state a rule.

$P = 2 \times (\ell + s)$ $\qquad$ $P = 2\ell + 2s$

We replace each variable ℓ and s with the given side lengths.

When we replace a variable with a number, we *substitute*.

$P = 2 \times (11 + 6)$ $\qquad$ $P = 2(11) + 2(6)$
$= 2 \times 17$ $\qquad$ $= 22 + 12$
$= 34$ $\qquad$ $= 34$

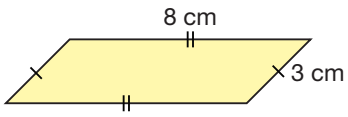
The perimeter of this parallelogram is 34 cm.

We can check by adding the lengths of the 4 sides:
11 cm + 6 cm + 11 cm + 6 cm = 34 cm
This is the same as the answers we got using the formulas.

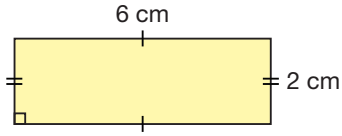
Practice

1. Find the perimeter of each polygon.

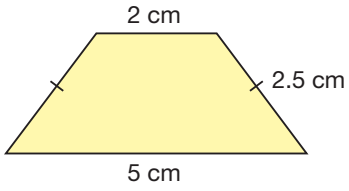
a)



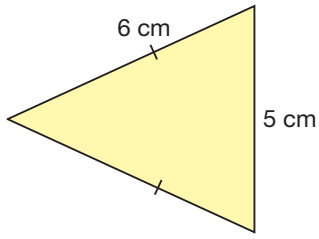
b)



c)



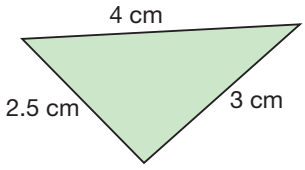
d)



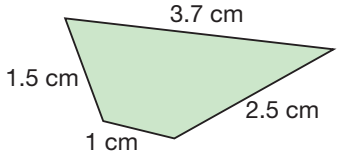
2. Describe the strategy you used to find the perimeter of each polygon in question 1.

3. Find the perimeter of each polygon.

a)



b)

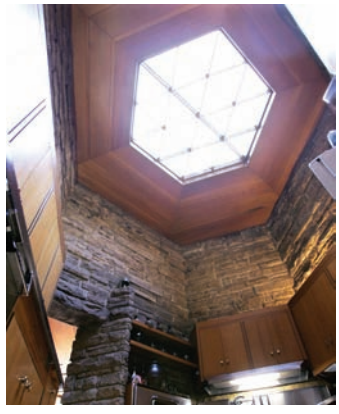


Can you write a rule to find the perimeter of each of these polygons? Why or why not?

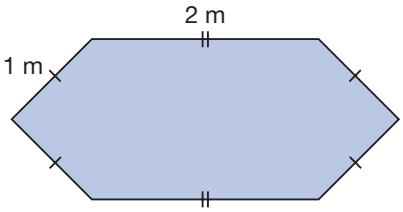
4. Use Pattern Blocks like those below.

Write a rule to find the perimeter of each Pattern Block.

5. Aldo wants to install a skylight in the roof of his house. The base of the skylight is a regular hexagon with side length 40 cm. What is the perimeter of the base of the skylight? Give your answer in metres.
Which strategy did you use to find out?



6. Winnie is building a hexagonal storage box.
Here is a drawing of the top of the box.

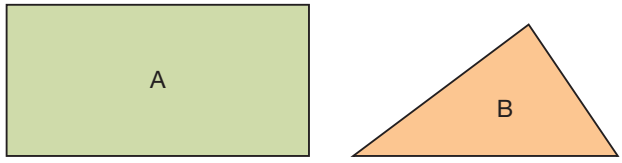


a) Write a rule to find the perimeter of the top of the box.

b) Write the rule as a formula.

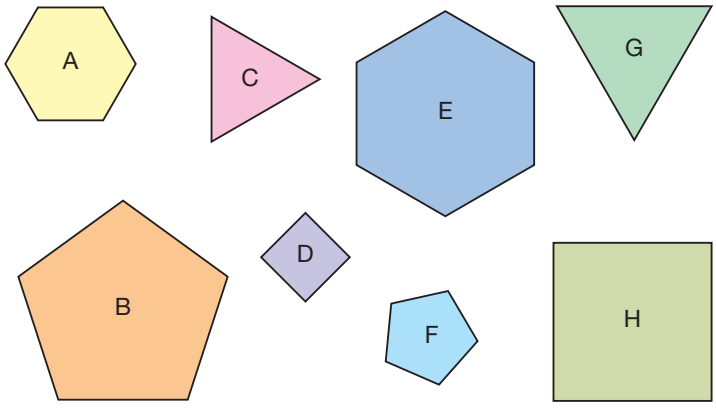
c) What is the perimeter of the top of the box?

7. **a)** Find the perimeter of each polygon.



b) Suppose the side lengths of each polygon are doubled.
What would happen to each perimeter? Explain.

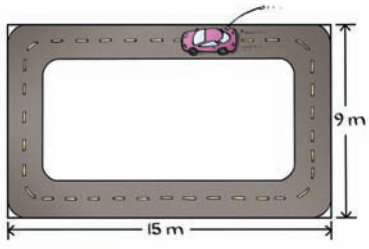
8. Your teacher will give you a large copy of these regular polygons.



a) Find and record the perimeter of each polygon.

b) How is the perimeter of a regular polygon related to the number of its sides?
Write a formula to find the perimeter of a regular polygon.

9. Saki has a remote control car. She enters her car in a race.
The track is close to rectangular.



a) Use a formula to find the perimeter of the track.

b) Suppose the car completes 8 laps.
How far did the car travel?

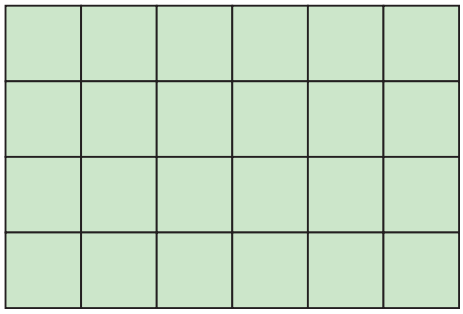
Reflect

How are the side lengths of a polygon and its perimeter related?
Use examples to explain.

LESSON 8

Area of a Rectangle

What is the area of this rectangle?
How did you find out?



Explore

You will need 1-cm grid paper.

- Draw a 2-cm by 3-cm rectangle.
  Find the area of the rectangle.
- Suppose the length of the rectangle doubles.
  Predict the area of the new rectangle.
  Check your prediction.
- Suppose the width of the original rectangle doubles. Predict the area of the new rectangle.
  Check your prediction.
- Suppose both the length and the width double.
  Predict the area of the new rectangle.
  Check your prediction.
- How does the area of each new rectangle compare to the area of the original rectangle?
- Write a rule to calculate the area of a rectangle.
  Write the rule as a formula.
  Use the formula to check the area of the rectangles you drew.

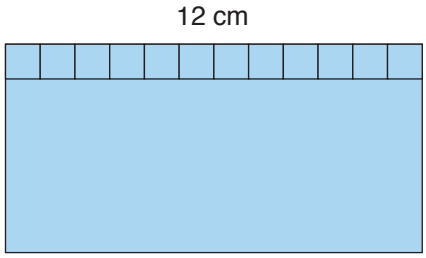
Show *and* Share

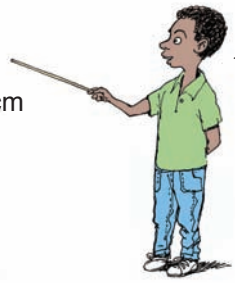
Share your work with another pair of students.
Compare your formulas.
What do you think happens to the area of a rectangle when the length triples? The width triples? Both the length and the width triple?
How could you use your formula to find out?

Connect

We can find a shortcut for calculating the area of a rectangle.

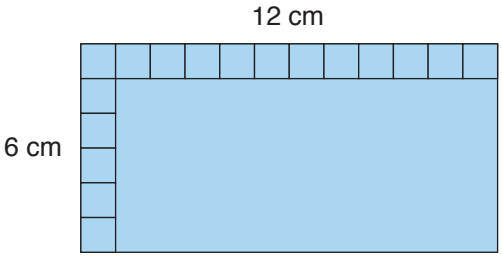
Measure the length of the rectangle.





The length tells how many 1-cm squares fit along it. The length is 12 cm. So, twelve 1-cm squares fit along the length.

Measure the width of the rectangle.

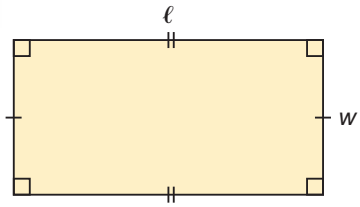


The width tells how many rows of 1-cm squares fit in the rectangle. The width is 6 cm, so there are 6 rows.

Multiply the length by the width.
$12 \times 6 = 72$
So, the area of the rectangle is 72 cm^2.

To find how many 1-cm squares fit in the rectangle, we multiply the length of a row by the number of rows.

We can write this rule:
To find the area of a rectangle, multiply the length by the width.

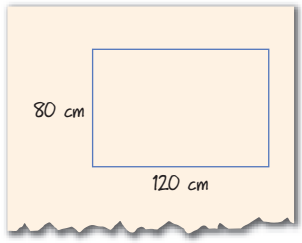


This rule can be expressed as a formula.

Area = length × width
$A = \ell \times w$

We use: A to represent area, ℓ to represent length, and w to represent width.

Edmond built a dog crate for his dog.
The floor of the crate is a rectangle.
The dimensions of the floor are 80 cm by 120 cm.



➤ You can use the formula for the area of a rectangle to find the floor area of the crate.

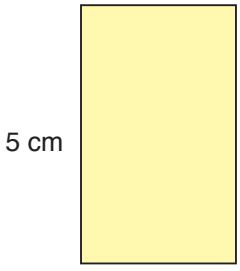
$A = \ell \times w$
$= 120 \times 80$
$= 9600$

The floor area of the crate is 9600 cm^2.

Practice

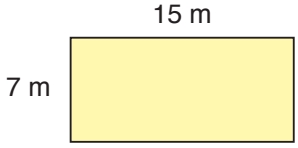
1. Find the area of each rectangle.

a)



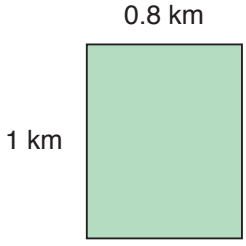
b)

c)

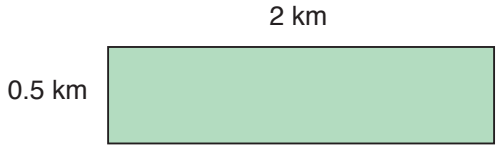


2. Which rectangle below do you think has the greatest area?
Estimate first. Use a formula to check.
Order the areas from least to greatest.
How does the order compare with your prediction?

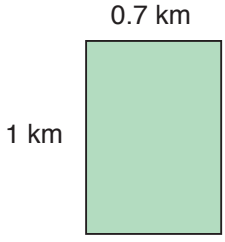
a)



b)



c)



3. Copy and complete this chart.

Rectangle	Length (cm)	Width (cm)	Area (cm^2)
A	7	5	?
B	?	6	12.6
C	3	?	13.5
D	5.3	7	?

Which strategy did you use to find the missing number each time?

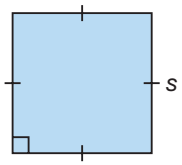
4. Matt's dog has a rectangular dog run.
The length of the dog run is 8 m. The total area enclosed is 56 m^2.
How wide is the dog run? Draw a diagram.
How can you use a number sentence to show your thinking?

5. Lena used 36 m of fencing to enclose a rectangular vegetable garden on her farm in Battleford, Saskatchewan.
   a) Sketch some possible rectangles and label their side lengths. What is the area of the enclosed section in each case?
   b) How many different answers can you find?

6. A banner for the Vancouver 2010 Olympics has length 226 cm and width 72 cm. What is the area of the banner?

7. Hailey bought a can of stain. The stain will cover 50 m^2 of fencing. The fence has height 2 m.
What length of fencing can Hailey stain before she runs out of stain? How did you find out?

8. A square has side length *s*.



Write a formula for the area of a square.

9. The Festival du Voyageur is a winter festival that takes place in St. Boniface, Manitoba, each February.
The festival's logo contains a red rectangle.
Suppose the logo is enlarged so the rectangle has width 4 cm and area 28.8 cm^2.
What is the length of the rectangle?
How did you find out?

10. Rectangle A has area 40 cm^2 and length 8 cm.
The area of Rectangle B is one-half the area of Rectangle A.
The rectangles have the same length.
What is the width of Rectangle B?

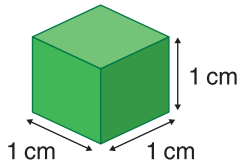
Reflect

When might you use the formula for the area of a rectangle outside the classroom?

LESSON 9

Volume of a Rectangular Prism

A centimetre cube has a length, width, and height of 1 cm.
What is its volume?



Explore

You will need 2 empty boxes and centimetre cubes.

- Choose one box.
  Estimate how many centimetre cubes the box can hold.
- Fill the bottom of the box with one layer of cubes.
  How many cubes are in that layer?
  How many layers can fit in the box?
  How do you know?
- How many cubes can the box hold altogether?
  Describe how you found your answer.
  Record your answer on the box.
- Without filling it completely, find how many cubes the second box can hold.
  Describe the strategy you used.
  Use cubes to check your answer.

Show *and* Share

Share the boxes you used with the class.
How can you find the volume of a box without filling it completely?
Will your answer be exact? Explain.
How can you find the volume of a box without using cubes?

Connect

A rectangular prism is 10 cm long, 5 cm wide, and 6 cm high.

The length is 10 cm.
It is 1 row of 10 cubes.
Volume of 1 row = 10 cm^3

The width is 5 cm.
Five rows of 10 cubes
make 1 layer of 50 cubes.
Volume of 1 layer = 5 × 10 cm^3
= 50 cm^3

The height is 6 cm.
Six layers of 50 cubes make
a volume of 300 cubes.
Volume of 6 layers = 6 × 50 cm^3
= 300 cm^3

We can use the descriptions above to develop a formula for the volume of a rectangular prism.

Volume in cubic centimetres
= number of 1-cm cubes in each layer × number of layers

The number of cubes in each layer is the area of the base of the prism. It is the length times the width.

The number of layers is the height of the prism.

So, Volume = base area × height

Another way to write the formula is:

Volume = length × width × height

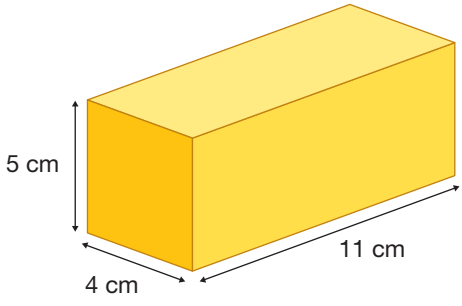
$V = \ell \times w \times h$

We use: V to represent volume, ℓ to represent length, w to represent width, and h to represent height.

➤ We can use the formula to find the volume of a rectangular prism 11 cm long, 4 cm wide, and 5 cm high.

Volume = $\ell \times w \times h$
= 11 cm × 4 cm × 5 cm
= 44 cm^2 × 5 cm
= 220 cm^3

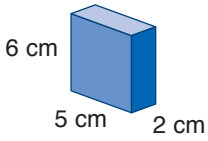
The volume of the prism is 220 cm^3.



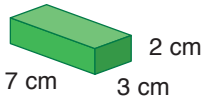
Practice

1. Find the volume of each rectangular prism.

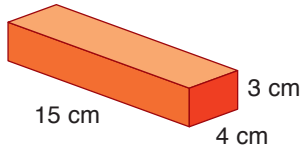
a)



b)



c)



2. Estimate, then calculate, the volume of a rectangular prism with these dimensions.

	Length (cm)	Width (cm)	Height (cm)
a)	6	2	2
b)	9	4	7
c)	18	9	12
d)	30	15	6

3. A dog box is built to fit in the back of a pick-up truck. It is used to transport sled dogs and supplies to a race. A dog box that holds 3 dogs is 117 cm long, 97 cm wide, and 61 cm tall. Each dog compartment is 38 cm long, 97 cm wide, and 46 cm tall.

a) What is the volume of each dog compartment?

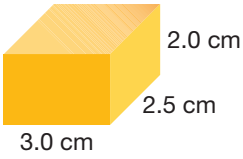
b) What is the volume of the dog box that is not used to hold dogs? How did you find out?

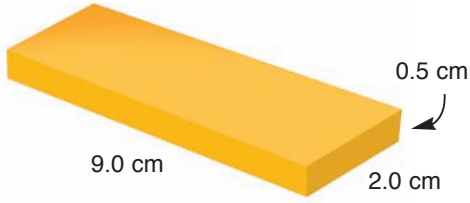
4. During the buffalo hunt, the Métis used a Red River cart to carry buffalo meat and fur. The cart was made of wood and was usually pulled by oxen. The top of this cart has the shape of a rectangular prism with volume 1 350 000 cm^3. The area of its base is about 13 500 cm^2. About how high is the top of the cart? Which strategy did you use to find out?

5. A rectangular prism has volume 90 cm^3.
The prism has length 9 cm and width 5 cm.
What is its height? How do you know?

6. A rectangular prism has volume 192 cm^3.
   a) The prism is 16 cm high. What is the area of its base?
   How do you know?
   b) What other possible measurements of height and base area could the rectangular prism have?
   What strategy did you use to find out?

7. Canada's Food Guide recommends that we eat 2 to 4 servings of dairy products every day.
   a) This piece of cheese is 1 serving of dairy products.
   What is its volume?





   b) Is the block of cheese at the right more or less than 1 serving? How do you know?

8. Each block in a child's set of building blocks is 15 cm long, 10 cm wide, and 5 cm high.
Suppose you put the blocks in a box that is 50 cm long, 35 cm wide, and 30 cm high.

   a) What is the volume of each block? Of the box?
   b) Suppose you only consider the volume.
   How many blocks would you expect to fit in the box?
   c) Suppose you arrange the blocks neatly in layers.
   How many different ways can you layer the blocks?
   How many blocks fit in the box each way?
   d) Compare your answers to parts b and c.
   Explain any differences.
   e) Which is the best way to pack the blocks? Why?

Reflect

Explain why the volume of a rectangular prism is the product of its length, width, and height. Include a diagram in your explanation.

Beat the Clock!

Your teacher will give you copies of 12 *Draw* cards,
12 *Explain* cards, and 12 *Size It Up* cards.
You will need scissors and a timer.
The goal of this game is to complete the most tasks and
to get the most cards.
Form 2 teams. Decide who will be Players A and B on each team.
For regular play, use 2-min timers.
For advanced play, use 1-min timers.

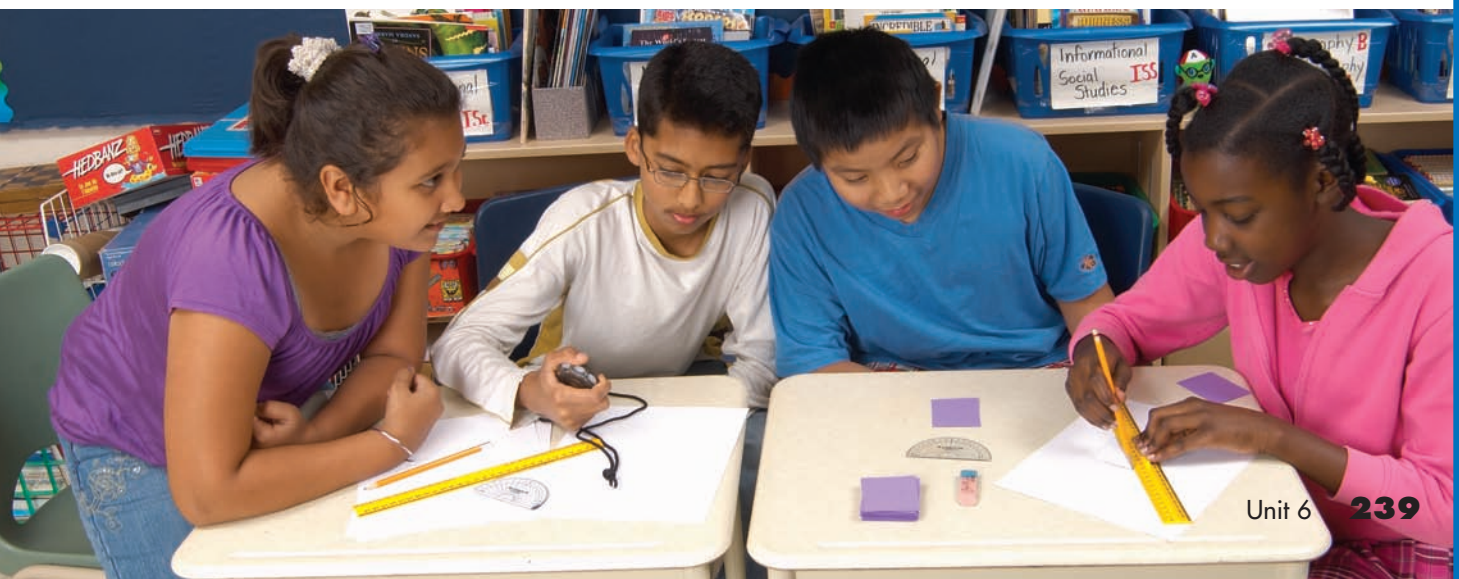
➤ Cut out the cards.
Shuffle the cards and place them face down in a pile.

➤ Teams take turns. Player A draws a card.
- If the card is a *Draw* card, Player A draws the shape.
  Player B guesses the shape.
- If the card is an *Explain* card, Player A describes the attributes
  of the shape. Player B guesses the shape.
- If the card is a *Size It Up* card, Players A and B work together
  to find the perimeter, area, or volume.

When the team is ready to begin, the other team starts the timer.

➤ A team keeps a card if the task was completed correctly.
If a team is not correct, the other team can steal the card by
giving the correct answer. If neither team is correct,
the card is returned to the pile.
A team completes as many cards as it can in the allotted time.

➤ Teams continue to take turns.
The team with more cards after an agreed time wins.

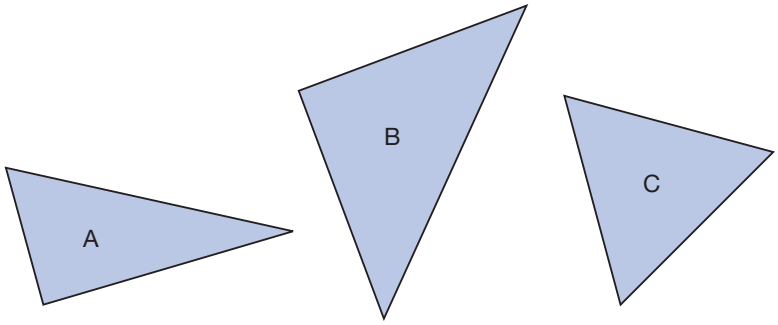


Unit 6 Show What You Know

LESSON

1 2

1. a) Name each triangle as scalene, isosceles, or equilateral.
Explain how you decided on each name.



b) Rename each triangle as acute, obtuse, or right.
Tell how you know.

3 5

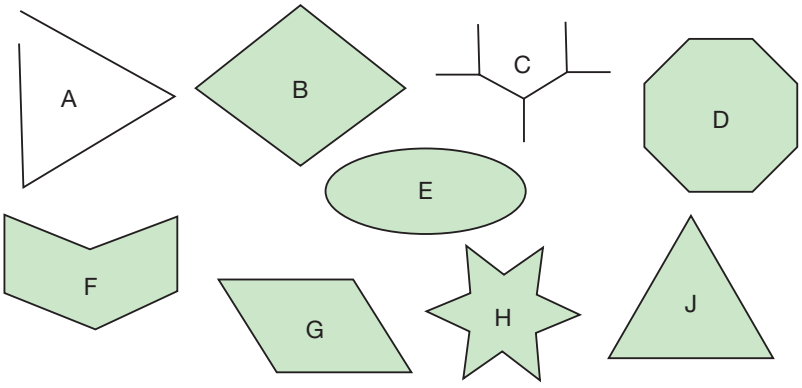
2. a) Use a ruler and a protractor.
Construct triangle RST: side RS is 5.6 cm, ∠R is 30°, and ∠S is 90°.
Sketch the triangle first.

b) What kind of triangle did you draw?
How else can you name the triangle?

c) Trace △RST.
Use the tracing to draw the triangle in a different orientation.
Explain how you know the two triangles are congruent.

4

3. a) Sort these shapes into sets of polygons and non-polygons.
Explain how you decided where to place each shape.



b) Sort the polygons in part a into sets of regular and irregular polygons.
Explain how you did this.

LESSON

5

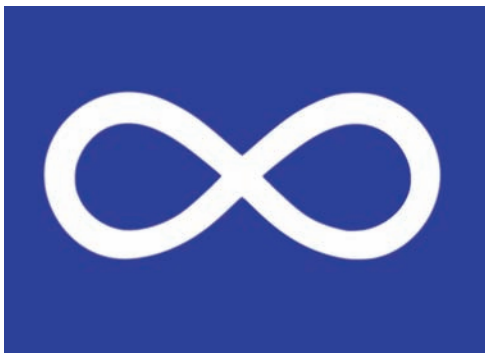
4. Draw a regular quadrilateral on square dot paper.
   a) What shape did you draw?
   b) Use measuring and superimposing to show that all angles are congruent and all sides are congruent.
   Show your work.

7

5. a) This sushi-platter pendant has the shape of a regular hexagon.
   The pendant has side length 1.9 cm.
   Calculate the perimeter of the pendant.
   Which strategy did you use?
   b) Write a formula to find the perimeter of any regular hexagon.
   Explain why the formula works.

8

6. The flag of the Métis Nation in Saskatchewan is rectangular.
   Suppose it has length 3 m and width 1.5 m.
   What is the area of the flag?
   How did you find out?



7 8

7. The top of Toby's desk has length 68 cm and width 50 cm.
   a) What is the area of the top of Toby's desk?
   b) Toby is working on a poster.
   The area of the poster is 2500 cm^2.
   Find 3 pairs of possible dimensions for the poster.
   How did you do this?
   Which dimensions are most likely?
   c) Can you tell if the poster fits on Toby's desk?
   Explain.

9

8. Estimate, then calculate, the volume of a rectangular prism with each set of dimensions.
   a) length 21 cm, width 19 cm, height 8 cm
   b) length 5 m, width 1.2 m, height 2 m

UNIT 6 Learning Goals

- ☑ construct and compare triangles
- ☑ describe and compare regular and irregular polygons
- ☑ develop formulas for the perimeters of polygons, the area of a rectangle, and the volume of a rectangular prism

Unit Problem

Puzzle Mania!

You will solve 2 puzzles, then design your own puzzle for others to solve.

Part 1

Triangle Detection

Matina was organizing the math lab.
She sorted triangles, then placed them in 3 sealed envelopes labelled A, B, and C.
Each envelope contains one type of triangle: equilateral, isosceles, or scalene.
Use the clues to solve the puzzle.

Clues

- Envelope B does *not* contain any regular polygons.
- Envelope A has some right triangles.
- All of the triangles in envelopes A and C have line symmetry.
- The triangles in envelope B do not have line symmetry.

Use the table to help.

Type of Triangle	Envelope A	Envelope B	Envelope C
Equilateral			
Scalene			
Isosceles			

Mark an X to eliminate a triangle from an envelope, and a ✓ to show a match.
Which type of triangle is in each envelope? Explain how you know.

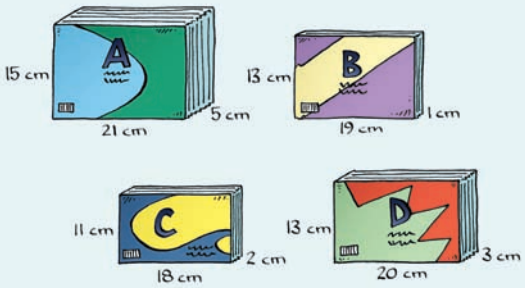
Books by Size

You will need a calculator.
Li used the dimensions of his 4 favourite books to create a puzzle.
He wrote each dimension to the closest centimetre.

Check List

Your work should show

- ☑ that you can use attributes to identify shapes
- ☑ completed tables and all calculations
- ☑ a clear explanation of how you solved each puzzle
- ☑ a clear explanation of how you designed and solved your puzzle



Use the clues and the table to match the books with their sizes. Show all calculations. Explain how you solved the puzzle.

Clues

- The front cover of *Stig of the Dump* has the least area.
- The volume of *The Little Prince* is less than that of *Stig of the Dump*, but the area of its front cover is greater.
- The front cover of *The Faraway Tree Collection* has the greatest perimeter.

Book	The Little Prince	Stig of the Dump	Swallows and Amazons	The Faraway Tree Collection
A				
B				
C				
D				

Part 2

Create your own geometry puzzle about regular and irregular polygons. Include at least 3 shapes and 3 clues. Make a table to record your reasoning. Explain how you created your puzzle. Solve your puzzle. Then trade problems with another pair of classmates and solve your classmates' puzzle.

Reflect on Your Learning

What have you learned about triangles and other polygons? Write about the different formulas you developed in this unit. Provide a real-world application for each formula.

Units 1–6 Cumulative Review

Unit

1

1. The table shows the input and output for a machine with two operations.
Identify the numbers and operations in the machine.

Input	Output
1	1
2	6
3	11
4	16
5	21

2

2. Use an integer to represent each situation.
Then use yellow or red tiles to model each integer.
Draw the tiles.

a) 13°C above zero **b)** 8 m below sea level
c) a withdrawal of $10 **d)** an apartment 7 floors above ground level

3

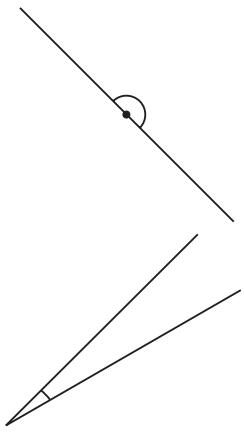
3. Estimate each product or quotient. Which strategy did you use?
Tell if your estimate is an overestimate or an underestimate.

a) 6.89×3 **b)** $621.45 \div 4$ **c)** 14.93×5 **d)** $41.625 \div 7$

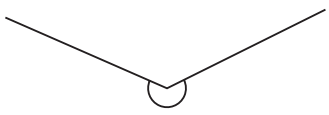
4

4. Measure each angle. Name each angle as acute, right, obtuse, straight, or reflex.

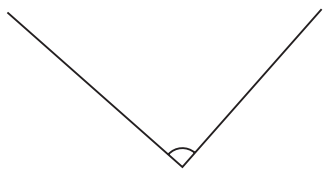
a)



b)

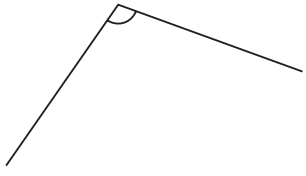


c)

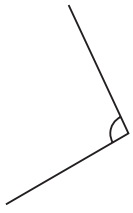


d)

e)



f)



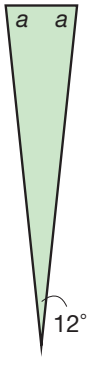
5. Use a ruler and a protractor. Draw an angle with each measure.

a) 35° **b)** 160° **c)** 310° **d)** 95°

6. A backgammon board contains 24 congruent triangles.
Here is one of the triangles.

a) Find the measures of the unknown angles without measuring.
Explain your strategy.

b) Check your answers by measuring with a protractor.



5

7. Place the numbers in each set on a number line.
Show how you did it. List the numbers from greatest to least.

a) $2\frac{1}{8}, \frac{5}{2}, \frac{9}{4}$ **b)** $\frac{3}{2}, \frac{5}{3}, 1\frac{5}{12}$

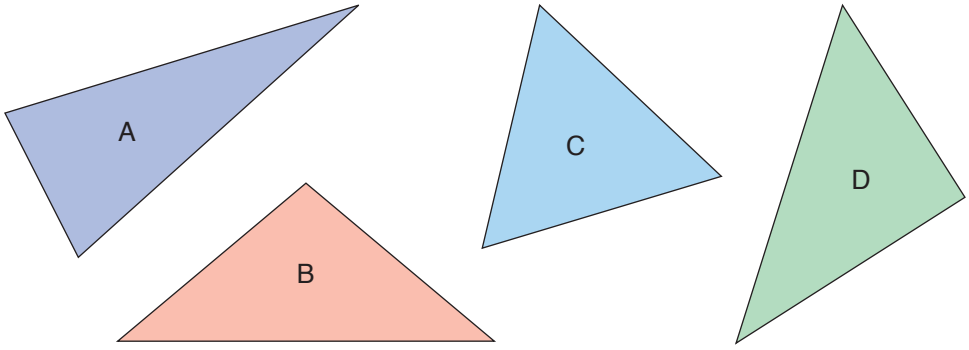
8. Chef Blanc uses 4 parts of oil for every 3 parts of vinegar to make a salad dressing for his restaurant in Hay River, NWT. Suppose he uses 12 parts of oil. How many parts of vinegar will he use?

9. Draw Base Ten Blocks or shade a hundredths grid to represent each amount.

a) $\frac{7}{50}$ **b)** 0.51 **c)** 29% **d)** 0.02 **e)** $\frac{3}{20}$ **f)** 9%

6

10. Use a ruler and a protractor.
Measure the sides and angles of each triangle.



a) Name each triangle by the number of equal sides.
Use the words: scalene, equilateral, isosceles

b) Name each triangle by the angle measures.
Use the words: acute, right, obtuse

11. Use dot paper. Draw two congruent regular polygons.
Trade shapes with a classmate.
Explain how you know your classmate's shapes are congruent.

12. **a)** This dinner plate is shaped like a regular octagon.
The side length of the octagon is 9.5 cm.
Calculate the perimeter of the dinner plate.
Which strategy did you use?

b) Write a formula that you could use to find the perimeter of any regular octagon. Explain why the formula works.